Subset problem.

$\underbrace{S = \{w_1, \ldots, w_n\}}_{\text{Sorted in incr.}}$. Find a subset of S, (say T) s.t

$$\sum_{i \in T} w_i \leq \underline{W}$$

and $\sum_{i \in T} w_i$ is maximized.

Want to pick a set $T = \{i_1, \ldots, i_k\}$ s.t $w_{i_1} + \cdots + w_{i_k} \leq W$ and $\sum_{j=1}^{k} w_{i_j}$ is maximized.

Case-1 : Choose w_n (if $w_n \leq W$).

We need to choose $\underbrace{\sum_{j=1}^{k'} w_{i'_j} \leq W - w_n}$.

and $\sum_{j=1}^{k'} w_{i'_j}$ is maximized.

$Opt(i, w)$: Optimal solution over $\{w_1, \ldots, w_i\}$ that satisfies $\sum_{p=1}^{|O|} w_{i_p} \leq w$.

In this case, we now seek $Opt(n-1, W - w_n)$.

Case-2 : $w_n \notin$ optimal solution.

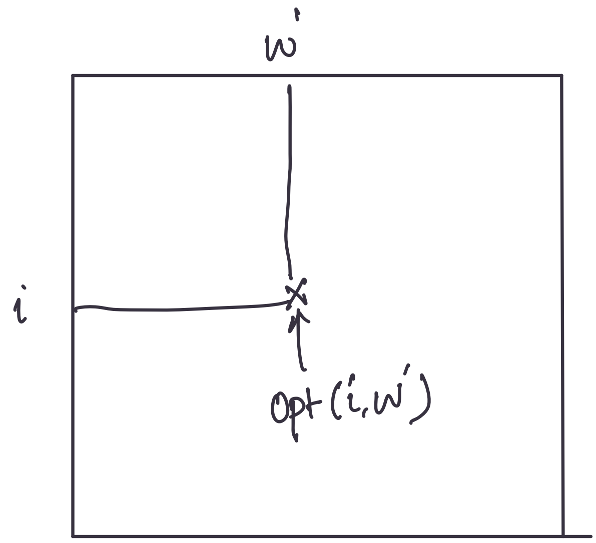
In this case, we seek $Opt(n-1, W)$.

$$Opt(n, W) = \max\{Opt(n-1, W),\ Opt(n-1, W-w_n) + w_n\}.$$

No. of subproblems $\le n \cdot W$

$Opt(i, w')$

$1, \dots, n$ $\qquad$ $0, 1, \dots, W$



$$opt(i, w') = \max\begin{cases} opt(i-1, w'), \\ opt(i-1, w'-w_i) + w_i \end{cases}$$

Base case:

$Opt(1, w')$ for $w' \in [0, W]$.

$w_1 \le w' \longrightarrow Opt(1, w')$ is w_1.

Else, $w_1 > w' \longrightarrow Opt(1, w') \longrightarrow$

0-1 Knapsack.

	Item 1	Item 2	...	Item n
Elems value	v_1	v_2		v_n
	w_1	w_2		w_n

Knapsack of total weight W.

Question: Fill your knapsack s.t the value of the contents is maximized.

In other words, find a subset of items say $i_1, \dots, i_k$

s.t $\sum_{j=1}^{k} w_{i_j} \le W$ and $\sum_{j=1}^{k} v_{i_j}$ is maximized.

Case-1: Item $n \in$ Optimal set.

$VOpt(n-1, W-w_n)$

Case-2: Item $n \notin$ Optimal set

$VOpt(n-1, W)$.

$$VOpt(n,W) = \max \{VOpt(n-1, W), VOpt(n-1, W-w_n) + v_n\}.$$

Matrix Chain Multiplication.

ABC

$A_{w_1 \times w_2}$ $w_0 \times w_1$

$B_{w_2 \times w_3}$ $w_1 \times w_2$

$C_{w_3 \times w_4}$ $w_2 \times w_3$

$\longrightarrow (AB) \times C$

$w_1 \times w_2 \times w_3$

$w_1 \times w_3 \times w_4$

$\left.\begin{matrix} \\ \\ \end{matrix}\right\} \begin{matrix} w_1 \times w_2 \times w_3 \\ + \\ w_1 \times w_3 \times w_4 \end{matrix}$

6000

$\longrightarrow A(BC)$

$w_2 \times w_3 \times w_4$

$+$

$w_1 \times w_2 \times w_4$

3000

$Opt(ABC)$ 1, 3

$\min \{ Opt(1,2) + Opt(3,3) + w_0 \times w_2 \times w_3,$

$Opt(1,1) + Opt(2,3) + w_0 \times w_1 \times w_3 \}$

$w_1 = 10$

$w_2 = 5$

$w_3 = 30$

$w_4 = 15$

$A_1 A_2 \cdots A_n \qquad A_i \longrightarrow w_{i-1} \times w_i$

$A_1 \rightarrow w_0 \times w_1$

Want to find an optimal sequence of multiplications.

$$Opt(i,i) = 0$$

$$Opt(i,j) = \min_{\substack{k \\ i \le k < j}} \left\{ Opt(i,k) + Opt(k+1,j) + w_{i-1} \times w_k \times w_j \right\}.$$