Dynamic Programming (Contd).

Question: Compute the shortest $s \rightsquigarrow t$ path in G.

Assumption: No negative cycles.

- Edges from a node.

Question: What is the max length of the shortest path from $s \to t$

↳ **Ans:** at most $n-1$.

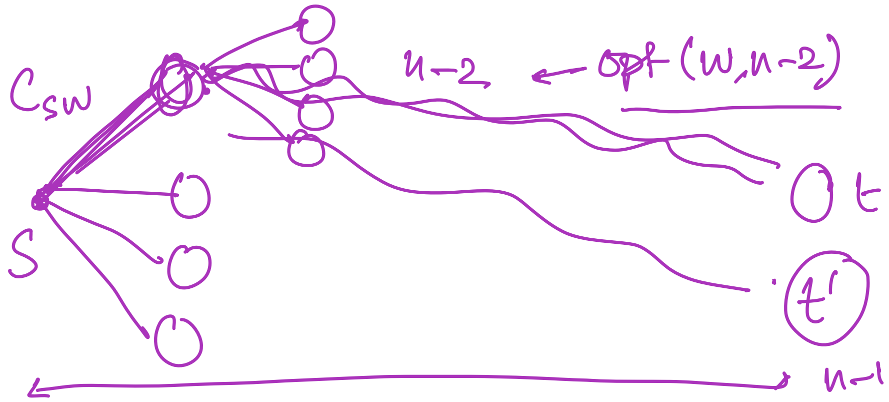
$Opt(v, \ell)$: min wt of all $v \rightsquigarrow t$ path of length at most ℓ.

$Opt(s, n-1)$: ← This is what we want.

Could happen
paths of length at most $n-2$ ✓

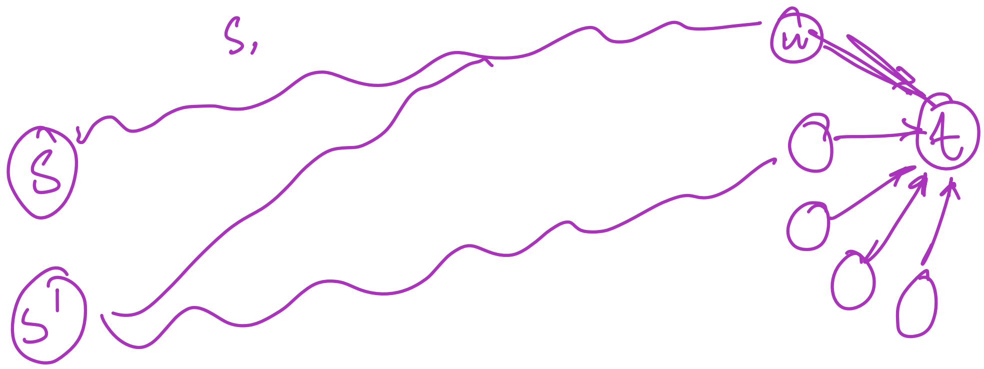
$$\min\{ opt(w, n-2) + C_{sw} \ : \ w \text{ s.t } (s,w) \in E \}$$

$$opt(s, n-1) = \min\Big\{ \{opt(s, n-2)\} \cup \{ opt(w, n-2) + C_{sw} : w \text{ s.t } (s,w) \in E \} \Big\}$$



$\widehat{opt}(\ell, v)$: min wt over all $s \rightsquigarrow v$ paths of length at most ℓ.

$$\widehat{opt}(n-1,t) = \min \begin{cases} opt(n-2,t) \\ \{opt(\underline{n-2},w) + \underline{c_{wt}} : (w,t) \in E\} \end{cases}.$$



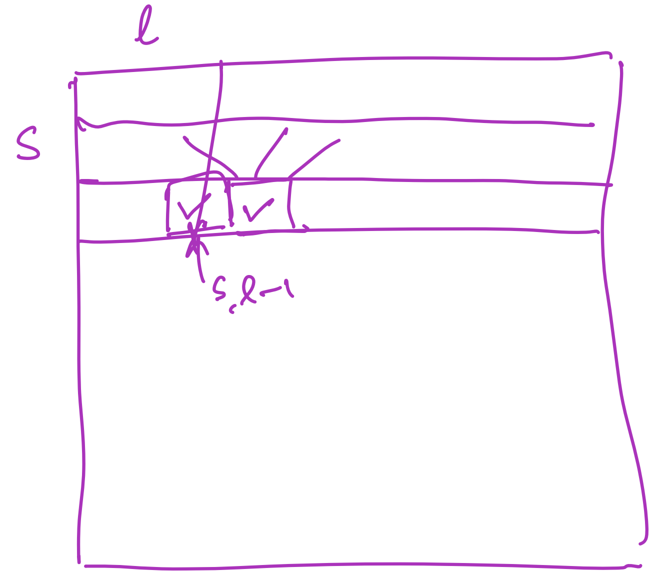
For each entry of the memoization, we consider all the neighbours of that node in consideration.

$\forall \ell \in [0, n-1]$

s, ℓ

$w, \ell-1$

$w, \ell-2$



$\underline{O(n^2+nm)}$ ✓.

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