Network Flows (contd.)

Suppose f is a feasible flow. ← Total flow = $|f|$.

$f(u \to v)$ are defined for $u \to v \in E$

Residual capacity

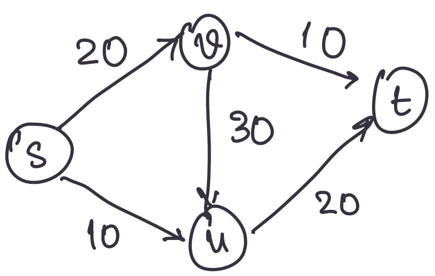
$$c_f(u \to v) = \begin{cases} c(u \to v) - f(u \to v) & \text{if } u \to v \in E \\ f(v \to u) & \text{if } v \to u \in E \\ 0 & \text{o/w} \end{cases}$$

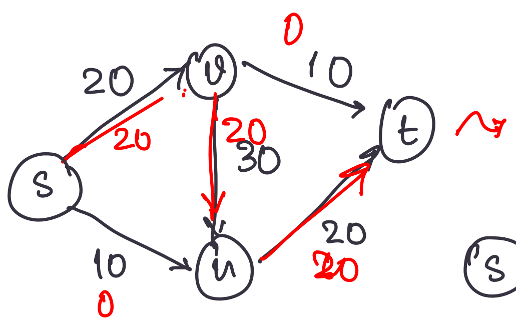
Residual graph is graph constructed with capacities $c_f(u \to v)$ $\forall\, u, v \in V(G)$.

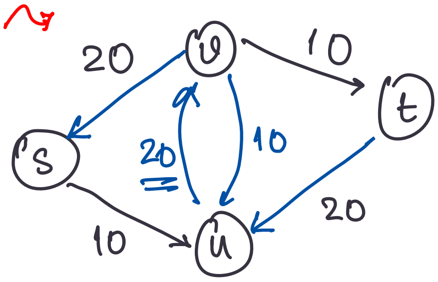
Find an $s \rightsquigarrow t$ path in the residual graph.

Let P be a $s \rightsquigarrow t$ path that gets picked.

$$F = \min_{u \to v \in P} \{ c_f(u \to v) \}$$







Available $s \rightsquigarrow t$ path: $s \xrightarrow[10]{} u \xrightarrow[\underset{10}{20}]{} v \xrightarrow[10]{} t$ P

min capacity = 10

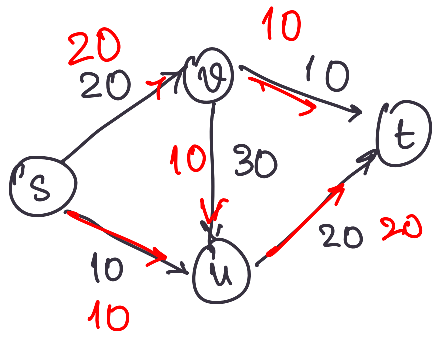
$s \to u \to$ $f'(s \to u) = 10$

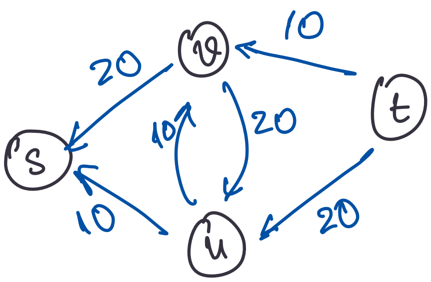
$f'(v \to u) = f(v \to u) - 10$

$f'(v \to t) = 10$

$f'(u' \to v') = f(u' \to v')$ for other edges.

Total flow $|f'| = 30$.





$S = \{s\}$, T

$$f'(u \to v) = \begin{cases} f(u \to v) + F & \text{if } u \to v \in E \cap P \\ f(u \to v) - F & \text{if } v \to u \in P \wedge u \to v \in E \\ f(u \to v). & \text{otherwise.} \end{cases}$$

Let f_{opt} is the flow when the algorithm terminates.

Let S be the set of vertices reachable from source. in res. graph

T be the rest of vertices in orig. graph.

$$0 = C_{f_{opt}}(\underbrace{\underset{\substack{\uparrow \\ \in S}}{u} \to \underset{\substack{\uparrow \\ \in T}}{v}}_{\in E}) = \underbrace{c(u \to v) - f_{opt}(u \to v)}_{\geq 0}$$

$$\sum_{u \to v \in \text{cut}} c(u \to v) = \sum_{u \to v \in \text{cut}} f_{opt}(u \to v)$$

$\Rightarrow |\text{max flow}| \geq |\text{min cut}|$.

Putting together $\rightarrow$ max flow = min cut.

Running time = No. of iterations $\rightarrow \leq$

$$\min\left\{ \sum_{w} c(s \to w), \sum_{u} c(u \to t) \right\}$$

- Search algo
- update on all edges.

Correctness:

- Show that $f'(u \to v) \in [0, c(u \to v)]$.
- Conservation is maintained.

1. Every augmentation of flow gives us a feasible flow
2. If there is no $s \to t$ path in $G_{residual}$, then we got the optimal value.