NP- hardness and NP completeness.

NP: Problems that can be efficiently verified

P: Problems that can be solved eff.

Circuit SAT: NP- hard

NP-hard: Problems when solved in PTIME

$\Downarrow$

NP = P.

Π is NP-hard if any problem in NP can be solved with Π' as a subroutine.

"**Subroutine**".

Reductions

$\Pi' \leq_p \Pi$

k-clique

Hamiltonian cycle

$\underbrace{\phi}_{\text{instance}} \in \underbrace{\Pi'}_{n} \longrightarrow \phi' \in$ Ckt-SAT.

$\overset{n}{G} \xrightarrow{T}$ Ckt-SAT problem $\overset{n^c}{C_G}$

Say N sized ckt has a f(N) time ago.

" G has a vertex cover of size at most k if and only if C_G has a satisfiable assignment".

" $|C_G|$ is at most poly(n) ".

$\Rightarrow$ Π' w.r.t insta G can be solved in

$\underline{|T|} + \underline{f(n^c)}$.

Further if $|T|$ is poly(n); and $f(N) = N^{O(1)}$
then $G \in \Pi'$ can be solved in $n^{O(1)}$.

"Ckt SAT is NP-hard"

$\Updownarrow$

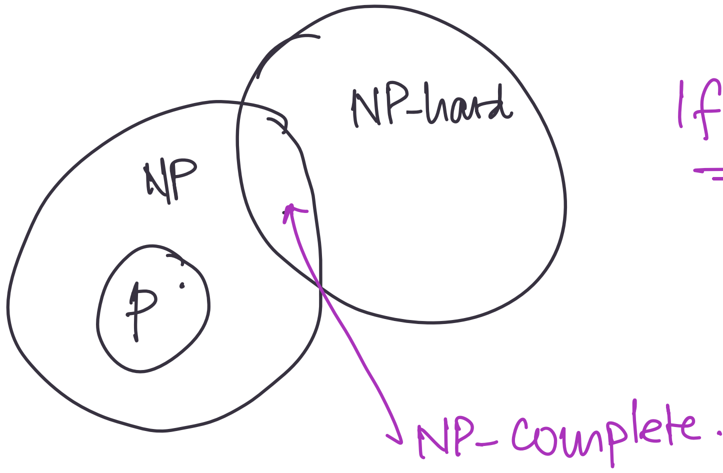
" Every instance of every problem in NP can be reduced to an instance of ckt satisfiability of size at most poly(|instance|), and reduction takes at most poly(|instance|) time".

- Traveling Salesperson problem
- Minimum vertex Cover $\equiv$ Maximum Independent Set
- k-vertex Cover, k-clique.

NP-complete: Problems that are NP-hard and also in NP.

" Ckt SAT is NP-complete"

To show $P \neq NP$:
- Suff to show that $\exists$ problem $NP \setminus P$ that admits no polytime algos.



If $P \neq NP$

Suff:

- Polynomial time algorithms for any NP-hard problem $\Rightarrow P = NP$.

- Proving that a NP-complete problem has no polytime algorithms $\Rightarrow P \neq NP$.

- Proving that a problem in $NP \setminus (P \cup \text{NP-complete})$ admits no polytime algo $\Rightarrow P \neq NP$

- Proving that a problem in $NP \setminus (P \cup \text{NP-complete})$ admits a polytime algo $\not\Rightarrow P = NP$

whereas

- Proving that a NP-complete problem has a polytime algorithm $\Rightarrow P = NP$.

3-SAT (3-CNF): A Boolean formula that can be expressed as a conjunction of clauses each of which is a disjunction of 3 literals.

$C_1 \wedge C_2 \wedge C_3 \cdots$

$C_i = (x_{i_1} \vee \bar{x}_{i_2} \vee x_{i_3})$

Reduction:

CktSAT

Boolean circuit C
$\overset{?}{\exists}$ a satisfying assignment

3SAT

Given a 3-CNF, is there a satisfying assignment.

$$S_1, S_2, \ldots, S_k \subseteq \{1, 2, \ldots, n\}$$

$$\left. L_i := \bigwedge_{j \in S_i} x_j = 0 \right\} \equiv \bigvee_{j \in S_i} \bar{x}_j \equiv 1$$

$$\bigwedge_{i=1}^{k} \left(\bigvee_{j \in S_i} \bar{x}_j \right)$$

Claim: 3-SAT is NP-hard.

- Maximum Independent set is NP-hard.

"Algorithms" — Jeff Erickson.
(UIUC)